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APRIL, 1859.

EDITED BY

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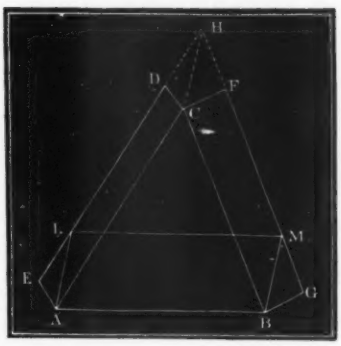
THE

MATHEMATICAL MONTHLY.

Vol. I....APRIL, 1859....No. VII.

PRIZE PROBLEMS FOR STUDENTS.

I.



On two sides $A\,C$ and $B\,C$ of any triangle $A\,B\,C$, let any parallelograms $A\,C\,D\,E$ and $B\,C\,F\,G$ be described. Let $E\,D$ and $F\,G$ produced meet in H; join $H\,C$, and through A and B draw $A\,L$ and $B\,M$ equal and parallel to $H\,C$. Join $L\,M$. It is required to prove that the parallelogram $A\,L\,M\,B$ on the side $A\,B$ is equal to the sum of the parallelograms on the sides $A\,C$ and $B\,C$.

Show also that the Pythagorean proposition is a particular case of this proposition.

II.

The area of a right-angled triangle is equivalent to the rectangle of the differences between the radius of the inscribed circle and the two shorter sides respectively; or the rectangle of the segments of the hypothenuse made by a perpendicular let fall upon it from the centre of the inscribed circle.

III.

The distance between two points A and B is a miles. A person starts at A and travels the first day one mth his distance to B; the second day he travels back one mth his distance to A; the third day he turns and travels one mth his distance to B, and so on. How far will he travel in n days, and how far will he be from A?

IV.

The volume of any right cone equals the product of its whole surface by one third the radius of the inscribed sphere.

V.

One pulley drives another by means of a belt; given the length of the belt l, the diameter D, of the larger pulley, the distance a between the centres of the pulleys; to find the diameter d, of the smaller pulley. Find also a simple approximate formula for the use of machinists.

REPORT OF THE JUDGES UPON THE SOLUTIONS OF THE PRIZE PROBLEMS IN NO. III. VOL. I.

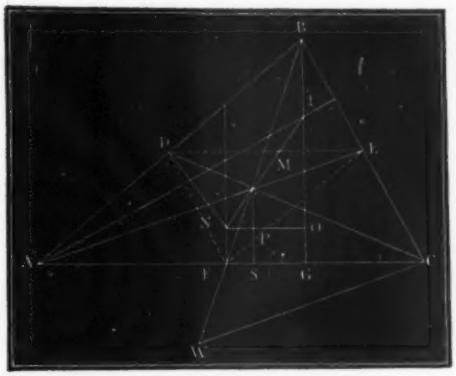
The first Prize is awarded to Asher B. Evans, of the Junior Class in Madison University, Hamilton, N. Y.

The second Prize is awarded to David Trowbridge, Perry City, Schuyler Co., N. Y.

PRIZE SOLUTION OF PROBLEM I.

"In every triangle, the perpendiculars dropped from the angles to the opposite sides meet in the same point; the perpendiculars erected to the middle of its sides meet in the same point; the lines drawn from the angles to the middle of the opposite sides meet in the same point. Prove that these three points are in the same straight line, and that their distances apart are in a constant ratio."

Let $A\,B\,C$ be any triangle; and let I, N, M be the three points. Produce $B\,F$ to W and draw $C\,W$ parallel to $A\,E$; then will $B\,M = M\,W$, since $B\,E = E\,C$. But the triangles $A\,M\,F$ and $F\,C\,W$ are equal; hence $M\,F = F\,W$. Therefore $B\,M : M\,F :: 2 : 1$. Again the triangle $A\,B\,C$ is similar to $D\,E\,F$, and the homologous sides are as 2:1. But $B\,I$ and $N\,F$ are



evidently homologous lines in these two triangles, and are also to each other as 2:1. Therefore $B\,M : M\,F :: B\,I : N\,F$. But the triangles $B\,M\,I$ and $N\,M\,F$ have the angle $M\,B\,I = N\,F\,M$; and since the sides containing these equal angles are proportional, these two triangles are similar. The sides $I\,M$ and $M\,N$ are therefore in constant ratio of 2:1; and since the angle $B\,M\,I = N\,M\,F$, and $B\,M\,F$ is a straight line, $I\,M\,N$ is a straight line also. The additional lines in the figure will suggest other ways of demonstrating the proposition.

This solution is by Asher B. Evans, with a slight abridgment.

PRIZE SOLUTION OF PROBLEM II.

"Prove that $\tan 9° = \sqrt{5} + 1 - \sqrt{5+2\sqrt{5}}$."

By the simple formulas of Trigonometry we have

$$\tan 5\theta = \frac{\tan\theta + \tan 4\theta}{1 - \tan\theta \tan 4\theta}; \quad \tan 4\theta = \frac{2\tan 2\theta}{1 - \tan^2 2\theta}; \quad \tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}.$$

Put $\tan\theta = x$; then

$$\tan 2\theta = \frac{2x}{1-x^2}, \quad \tan 4\theta = \frac{4x(1-x^2)}{(1-x^2)^2 - 4x^2},$$

$$\tan 5\theta = \frac{x(1-x^2)^2 - 4x^3 + 4x(1-x^2)}{(1-x^2)^2 - 4x^2 - 4x^2(1-x^2)} = 1, \text{ when } \theta = 9°.$$

By reduction

$$1 - 5x - 10x^2 + 10x^3 + 5x^4 - x^5 = 0.$$

Adding $20x^2 - 20x^3$ to both members of this equation it becomes

$$1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5 = (1-x)^5 = 20x^2(1-x).$$

Hence, dividing by $1-x$ we get

$$(1-x)^4 = 20x^2; \text{ or } (1-x)^2 = \pm 2x\sqrt{5};$$

and readily find that the five roots are

(1) $x = 1 = \tan 45°,$

(2) $x = 1 + \sqrt{5} + \sqrt{5+2\sqrt{5}} = \tan 81°,$

(3) $x = 1 + \sqrt{5} - \sqrt{5+2\sqrt{5}} = \tan 9°,$

(4) $x = 1 - \sqrt{5} + \sqrt{5-2\sqrt{5}} = \tan (-27°),$

(5) $x = 1 - \sqrt{5} - \sqrt{5-2\sqrt{5}} = \tan (-63°).$

The tan 9° is positive and less than unity, and (3) is the only root which fulfils these conditions. (2) and (3) are reciprocals, and therefore the corresponding arcs are complimentary. The same is true of (3) and (4).

This interesting solution is by DAVID TROWBRIDGE. The solution by E. P. AUSTIN involves the same method, but is less complete.

PRIZE SOLUTION OF PROBLEM III.

"Find the coefficient of x^n in the expansion of $\frac{ax+b}{(x-1)(x+2)(x-3)}$ into a series of ascending powers of x."

$$\text{Put } \frac{ax+b}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$$

$$= \frac{(A+B+C)x^2 - (A+4B-C)x - (6A-3B+2C)}{(x-1)(x+2)(x-3)}.$$

By the theory of indeterminate coefficients

$$A+B+C=0, A+4B-C=-a, 6A-3B+2C=-b,$$

and these equations give

$$A = -\frac{a+b}{6}, B = \frac{b-2a}{15}, C = \frac{3a+b}{10}.$$

Therefore by substitution we get

$$\frac{ax+b}{(x-1)(x+2)(x-3)} = \frac{a+b}{6} \times \frac{1}{1-x} + \frac{b-2a}{30} \times \frac{1}{1+\frac{x}{2}} - \frac{3a+b}{30} \times \frac{1}{1-\frac{x}{3}}.$$

Now develop each of the fractions involving x by division, and we readily find

$$\frac{a+b}{6} \pm \frac{b-2a}{30 \cdot 2^n} - \frac{b+3a}{30 \cdot 3^n}$$

to be the required coefficient; the sign of the middle term being + when n is even and — when it is odd.

This solution is by Asher B. Evans.

PRIZE SOLUTION OF PROBLEM IV.

"The equations (1) $y = ax + a + 1$, (2) $y = (a+1)x + b$, (3) $y = bx + 5$, are equations of straight lines in which a and b are positive integers, and the three lines all meet in one point. What values can a and b have, and what are the corresponding coördinates of the common points of the lines?"

From (1) and (2) $x = a + 1 - b$, and from (2) and (3) $(a + 1 - b)x = 5 - b$; and eliminating x from these equations we get $(a - b + 1)^2 = 5 - b$. Therefore

$$a = -1 + b \pm \sqrt{5-b};$$

which, with the given conditions, can only be satisfied by making $b = 1, 4, 5$.

The following sets of corresponding values are easily found:

$$\left.\begin{matrix} b=1 \\ a=2 \\ x=2 \\ y=7 \end{matrix}\right\} \quad \left.\begin{matrix} a=2 \\ b=4 \\ x=-1 \\ y=1 \end{matrix}\right\} \quad \left.\begin{matrix} a=4 \\ b=4 \\ x=1 \\ y=9 \end{matrix}\right\} \quad \left.\begin{matrix} a=4 \\ b=5 \\ x=0 \\ y=5 \end{matrix}\right\}.$$

For the last set of values lines (2) and (3) coincide. This solution is by E. P. Austin.

PRIZE SOLUTION OF PROBLEM V.

"When the angle between the coördinate axes is $\frac{\pi}{3}$; prove that $x + y - a = \sqrt{xy}$ is the equation of a circle, of which the radius is $\frac{1}{3}a\sqrt{3}$."

Let this equation be transformed to a system of rectangular coördinates, having the same origin and axis of x. The equations of transformation from oblique to rectangular coördinates are (see J. M. PEIRCE's Analytic Geometry)

$$x = \frac{x_0 + x_1 \sin\left(\frac{y}{x} - \frac{y_1}{x_1}\right) - y_1 \cos\left(\frac{y}{x} - \frac{y_1}{x_1}\right)}{\sin \frac{y}{x}}$$

$$y = \frac{y_0 + x_1 \sin \frac{y_1}{x_1} + y_1 \cos \frac{y_1}{x_1}}{\sin \frac{y}{x}}.$$

In this case $\frac{y_1}{x_1} = 0, \frac{y}{x} = \frac{1}{3}\pi, x_0 = 0, y_0 = 0$, and thee quations of transformation become $x = x_1 - y_1\sqrt{\frac{1}{3}}, y = 2y_1\sqrt{\frac{1}{3}}$. Substituting these values of x and y in the square of the given equation, it becomes

$$x_1^2 + y_1^2 - 2ax_1 - 2ay_1\sqrt{\tfrac{1}{3}} + a^2 = 0; \text{ or}$$

$$(x_1 - a)^2 + (y_1 - a\sqrt{\tfrac{1}{3}})^2 = \tfrac{1}{3}a^2;$$

which is the equation of a circle, of which the coördinates of the centre are $x_0 = a, y_0 = a\sqrt{\frac{1}{3}}$, and radius $= \frac{1}{3}a\sqrt{3}$. This solution is by ARTHUR WILKINSON.

JOSEPH WINLOCK,
CHAUNCEY WRIGHT,
TRUMAN HENRY SAFFORD.

PROPOSITIONS RELATING TO THE RIGHT-ANGLED TRIANGLE.

BY DAVID W. HOYT,
Adjunct Prof. of Mathematics and Mechanics in the Polytechnic College, Philadelphia.

1. PYTHAGOREAN PROPOSITION. This can be demonstrated by proving the square described on the hypothenuse, and the sum of the squares described on the other two sides, each equal to four times the triangle, added to the square of the difference of the two shorter sides. In the annexed figure $A\,O$ and $D\,L$ are each made equal to

BC. The rectangle $ONLD$ and the square $BCGF$ form the rectangle $BCPI$, which is equal to $ABMO$, or $RABC$; and $LEMN$ is the square of the difference of AB and BC. The remainder of the proof is sufficiently evident from the figure.

The above demonstration is deduced from the common method of cutting the squares BD and BG into five parts, which may be so combined as to form the square AK. This division is effected by drawing a line through B, parallel to AC, and erecting a perpendicular to AD at its extremity O.

2. Having given the hypothenuse, h, and the difference d, of the two shorter sides, the area of a right-angled triangle is expressed by $\frac{h^2-d^2}{4}$. This is merely a corollary of the preceding demonstration.

3. Having given the hypothenuse, h, and the radius of the inscribed circle, r, the area of a right-angled triangle is expressed by $hr + r^2$.

4. The area of a right-angled triangle is equivalent to the rectangle of the differences between the radius of the inscribed circle and the two shorter sides respectively; or the rectangle of the segments of the hypothenuse made by a perpendicular let fall upon it from the centre of the inscribed circle.

5. If a straight line be drawn from the vertex of either acute angle of a right-angled triangle to the centre of the inscribed circle, the square of this line, added to the rectangle of the opposite side and the hypothenuse, will be equivalent to the square of the hypothenuse.

6. The hypothenuse of a right-angled triangle is to the sum of the two lines drawn from the vertices of the two acute angles to the centre of the inscribed circle, as the difference of those lines is to the difference of the other two sides of the triangle.

7. The square of the diameter of a circle inscribed in a right-angled triangle is equivalent to twice the rectangle of the differences between the diameter and the two shorter sides respectively.

8. The rectangle of the lines drawn from the vertices of the two acute angles of a right-angled triangle to the centre of the inscribed circle, is to the rectangle of the hypothenuse and radius, as $\sqrt{2}$ to 1.

The above propositions can be demonstrated by any one having a knowledge of the principles contained in the first four books of LEGENDRE. They are given in the hope that they may be of service to those teachers who wish to make a practical application of the remarks contained in former numbers of this Monthly concerning the study of Geometry. The last six theorems are intimately connected with each other, and with the principle that the sum of the hypothenuse and diameter of the inscribed circle is equal to the sum of the other two sides.

PROPOSITIONS RELATING TO A PARTICULAR CONE.

BY DANIEL KIRKWOOD, LL. D.,
Prof. of Mathematics and Civil Engineering in Indiana University, Bloomington.

IN the fourth number of this Journal Professor SNELL states several curious properties of the cone whose slant height has to the radius of its base the ratio of 3 to 1. The following propositions in regard to the cone whose slant height is to the radius of its base in the ratio of $\sqrt{3}$ to 1 are equally interesting:

1. The right cone, whose slant height is to the radius of the base as $\sqrt{3}:1$, has *greater convex surface* than any other cone inscribed in the same sphere.

2. It has *greater volume* than any other cone inscribed in the same sphere.

3. Its base divides the circumscribed sphere into two segments such that the curve surface of one is *twice* that of the other.

4. Its curve surface is to that of the greater segment into which its base divides the circumscribed sphere, as the radius of the base is to the slant height.

5. Its volume is equal to that of the sphere, of which its altitude is the diameter

6. Its slant height is equal to the base of the *greatest parabola* that can be cut from it.

7. The area of such maximum parabola is to the curve surface of the lower segment of the sphere, as the diameter of a circle is to its circumference, or as $1 : \pi$.

8. The *curve surface* of its greatest inscribed cylinder is to the *base* of such cylinder, as the diagonal of a square is to the side, or as $\sqrt{2} : 1$.

9. Its centre of gravity is the centre of the circumscribed sphere.

10. Its altitude is to its slant height as $\sqrt{2} : \sqrt{3}$.

11. Its altitude is to the diameter of the circumscribed sphere, as $2 : 3$.

12. Its base is to that of the least cone that can be described about its circumscribed sphere, as $2^2 : 3^2$.

13. Its volume is to that of the circumscribed sphere, as $2^3 : 3^3$.

14. Its centre of gravity coincides with that of the *least cone* that can be described about its circumscribed sphere.

NOTES ON THE THEORY OF PROBABILITIES.

By Simon Newcomb, Nautical Almanac Office, Cambridge, Mass.

9. The theorem given in § 8 may be extended to any number of independent events. Let there be three events, E_1, E_2, E_3, of which

the respective probabilities are p_1, p_2, p_3. Call p' the probability of the compound event $(E_1 E_2)$. Then by the theorem the probability that this compound event and the event E_3 will *both* occur will be $p' \times p_3 = p_1 \times p_2 \times p_3$, since $p' = p_1 \times p_2$. By repeating the same process it may be shown that in general if there are n events, of which the probabilities are p_1, p_2, $p_3 \ldots . p_n$, the probability of the concurrence of all these events is the continued product $p_1 \times p_2 \times \ldots . p_n$. If the p's are mostly small fractions, and n large, this product will be very small.

10. If the events are not entirely independent, that is, if the probability of the second is affected by the occurrence or non-occurrence of the first; then, the probability of the compound event is equal to the probability of the first multiplied into the probability of the second *on the supposition that the first occurs.*

Let c be the whole number of possible cases, — let p of these cases be favorable to the first event, and of these p cases let q be favorable to the second. The probability that both events will occur is then $\frac{q}{c}$. But $\frac{p}{c}$ is the probability of the first event, and if this event occurs, the probability of the second becomes $\frac{q}{p}$, the product of which into $\frac{p}{c}$ gives $\frac{q}{c}$, which proves the proposition.

It may be remarked that this theorem applies also to independent events, and includes that of § 8, the probability of the second event being then the same, whether the first event be supposed to occur or fail.

Example. What is the probability that out of a bag which contains two white and two black balls, a white ball will be drawn at the first drawing and a black ball at the second? In this case, if the first event occur, the probability of the second will be $\frac{2}{3}$, since two out of the three balls remaining will be black. The probability of the first event being $\frac{1}{2}$, the probability of the compound event will be $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$.

11. *Conflicting events.* When a system of events are so connected that one *must* and only one *can* occur, they are called a *system of conflicting events.*

The sum of the probabilities of a system of conflicting events is equal to unity; since, every separate case being favorable to one of them, and to one only, the sum of the numerators of the different fractions expressing the probabilities (§ 6) is equal to the common denominator.

12. *Value of expectation.* Let there be n men, q of whom to be determined by lot, are certain of receiving the sum s each. What is each man's chance worth? If each man were to sell out his chance, the purchaser should give in all the sum $q\,s$ which he would then be sure of receiving, and since each man's chance is equal, he ought to give each man $\frac{q}{n}s$. But $\frac{q}{n}$ is the probability for each man that he will receive the sum s, and the value of his chance is therefore equal to the sum that he may receive multiplied by the probability that he will receive it.

PROBLEMS IN PROBABILITIES.

A has the reputation of telling the truth as often as three times in four; B as often as four times in five; and C as often as five times in seven. When A and B agree in affirming what C denies, what is the probability that A and B tell the truth? — Communicated by Professor Root.

A person goes on throwing a common die until he throws an ace; at whatever throw this occurs (the nth) he is to receive the nth of a dollar. What is the value of his expectation? — Communicated by Asher B. Evans.

DEMONSTRATION OF THE PROPOSITIONS RELATING TO THE CONE AND SPHERE.*

BY E. S. SNELL,
Prof. of Mathematics in Amherst College, Amherst, Mass.

1. To find the greatest cone within a given superficies. Let $r =$ radius of base, and $s =$ slant height. Then, perimeter of base $= 2\pi r$; area of base $= \pi r^2$; convex surface $= \pi r s$. Therefore the given area

$$A = \pi r^2 + \pi r s. \quad \therefore s = \frac{A}{\pi r} - r:$$

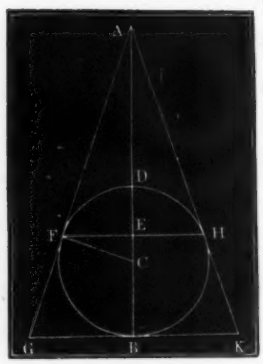
and the volume

$$v = \frac{\pi r^2}{3}(s^2 - r^2)^{\frac{1}{2}} = \frac{\pi r^2}{3}\left(\frac{A^2}{\pi^2 r^2} - \frac{2A}{\pi}\right)^{\frac{1}{2}},$$

which is to be a maximum. For convenient differentiation, square this function, reject the constant factors, and we have $A^2 r^2 - 2\pi A r^4$. Its differential coefficient

$$2A^2 r - 8\pi A r^3 = 0. \quad \therefore r = \tfrac{1}{2}\sqrt{\tfrac{A}{\pi}}.$$

But, $s = \frac{A}{\pi r} - r = \frac{A}{\pi} \div \frac{1}{2}\sqrt{\frac{A}{\pi}} - \frac{1}{2}\sqrt{\frac{A}{\pi}} = \frac{3}{2}\sqrt{\frac{A}{\pi}} = 3r.$



Therefore the slant height equals three times the radius of the base. This proves the truth of proposition 1.

2. To find the cone of least *entire surface*, circumscribing a given sphere.

Let the figure represent the section of the cone and sphere through the axis. Let $FC = a$, and $AC = x$. Hence, $AB = a + x$, and $AF = (x^2 - a^2)^{\frac{1}{2}}$. AFC and ABG are similar triangles;

---

* In No. IV. p. 121, 122.

$\therefore (x^2 - a^2)^{\frac{1}{2}} : x :: x + a : A\,G = \frac{x\,(x+a)}{(x^2-a^2)^{\frac{1}{2}}} =$ slant height.

$\therefore (x^2 - a^2)^{\frac{1}{2}} : a :: x + a : B\,G = \frac{a\,(x+a)}{(x^2-a^2)^{\frac{1}{2}}} =$ radius of base.

$\therefore \frac{2\,\pi\,a\,(x+a)}{(x^2-a^2)^{\frac{1}{2}}} =$ perimeter of base.

$\therefore \frac{\pi\,a^2\,(x+a)^2}{x^2-a^2} = \frac{\pi\,a^2\,(x+a)}{x-a} =$ area of base.

$\therefore \frac{x\,(x+a)}{(x^2-a^2)^{\frac{1}{2}}} \times \frac{\pi\,a\,(x+a)}{(x^2-a^2)^{\frac{1}{2}}} = \frac{\pi\,a\,x\,(x+a)}{x-a} =$ convex surface.

$\therefore \frac{\pi\,a^2\,(x+a)}{x-a} + \frac{\pi\,a\,x\,(x+a)}{x-a} = \frac{\pi\,a\,(x+a)^2}{x-a} =$ whole surface, which is to be a minimum. The differential coefficient is

$$\pi\,a\left(\frac{2\,(x+a)\,(x-a) - (x+a)^2}{(x-a)^2}\right) = 0;\ \text{and}\ x = 3\,a.$$

Therefore $A\,B = 4\,a =$ twice the height of the sphere. Substituting $3\,a$ for x in the expression for surface, $\frac{\pi\,a\,(x+a)^2}{x-a}$, we have $8\,\pi\,a^2$. But the surface of the sphere is $4\,\pi\,a^2$; therefore, the entire surface of the required cone is *twice* that of the sphere. Slant height : rad. of base $:: x : a :: 3\,a : a :: 3 : 1$. This cone, therefore, has the same form as that in the first demonstration; and thus, propositions 2, 4, and 5 are proved.

3. To find the cone of least *volume*, circumscribing a given sphere.

As before, area of base $= \frac{\pi\,a^2\,(x+a)}{x-a}$, and height of cone $= x + a$. Therefore the volume $= \frac{\pi\,a^2\,(x+a)^2}{3\,(x-a)}$, which is to be a minimum. This function differs from that for the entire surface, only in having the constant factor $\frac{1}{3}\,a^2$ in place of a. Hence, as before, $x = 3\,a$. This cone is, therefore, identical with that in the second demonstration. The volume, $\frac{\pi\,a^2\,(x+a)^2}{3\,(x-a)} = \frac{16\,\pi\,a^4}{6\,a} = \frac{1}{3}\,\pi\,(2\,a)^3$. But the volume of the sphere $= \frac{1}{6}\,\pi\,(2\,a)^3$; therefore, the volume of the

cone is *twice* that of the inscribed sphere. Thus have been proved propositions 3 and 6.

4. To find in what ratio the spheric surface is divided by the circle of contact.

AC has been proved equal to $3CF$. But, $AC : CF :: CF : EC :: 3 : 1$; therefore $BE : ED :: 3 + 1 : 3 - 1 :: 2 : 1$; and the surfaces, $FBH : FDH :: 2 : 1$; which is proposition 7.

5. Proposition 8 was accidentally stated wrong; it should have been, "The surface of the inscribed sphere is *twice* the area of the base of the cone." To prove it, we have only to compare the expressions for the two in the second demonstration.

The area of the base $= \frac{\pi a^2 (x + a)}{x - a} = 2\pi a^2$. But the surface of the sphere $= 4\pi a^2$, or twice the base of the cone.

Remark. Instead of eliminating s from the volume, as Professor SNELL has done, we may prove proposition 1 as follows:

Since the whole surface $= \pi r^2 + \pi r s =$ constant, and the volume $= \frac{\pi r^2}{3} (s^2 - r^2)^{\frac{1}{2}} =$ maximum, the derivatives of both functions must equal zero. Removing constant factors and radicals for convenience, we have

$$r^2 + r s = \text{constant},$$
$$r^4 s^2 - r^6 = \text{maximum}.$$

The derivatives are

$$2r + s + r D_r s = 0,$$
$$4 r^3 s^2 + 2 r^4 s D_r s - 6 r^5 = 0.$$

Eliminating $D_r s$ from these equations, we obtain

$$s^2 - 2 s r - 3 r^2 = 0;$$

and, therefore, $s = 3r$, or $-r$; one root corresponding to a maximum and the other to a minimum.

JAMES CLARK, Esq., of Wayne, Maine, sent us demonstrations of the above propositions, calling attention to the oversight in proposition 8, which Professor SNELL has corrected.

TO DESCRIBE A CIRCLE TANGENT TO THREE GIVEN CIRCLES.

BY H. A. NEWTON,
Professor of Mathematics, Yale College, New Haven, Conn.

THE method of transformation of curves by reciprocal radii vectores* affords a ready solution of the problem of drawing circles tangent to three given circles, provided two of the given circles cut each other.

This method is as follows. Let any point P be taken and called the *pole*, and any arbitrary square be taken as a superficial unit.

Two points, A and B, are said to be *reciprocal* to each other if they are on the same straight line through P and the rectangle $PA.PB$ is equal to the assumed unit square.

Two curves are said to be *reciprocal* to each other when the reciprocal to each point of the one is on the other.

The reciprocal of a straight line is a circumference that passes through the pole.

Let AB (Fig. 1) be a straight line, and P the pole. Let fall PA perpendicular on AB, and let C be the reciprocal to A. On CP as a diameter describe a circle CFP. From D any point of AB draw DP and take E the reciprocal of D. E is in the circumference CFP. For, join CE; then since by hypothesis $PA.PC = PD.PE$, that is $PA:PD :: PE:PC$, the triangles PEC and PAD are similar, and the angle $PEC = PAD$ a right

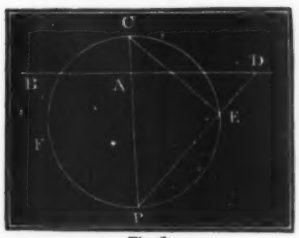


Fig. 1.

* Lionville XII. 265, and XIII. 209. Salmon's Higher Plane Curves, p. 239. Serret Méthodes en Géométrie, p. 21. Camb. and Dub. Math. Journal, Feb. 1853. Quarterly Jour. of Pure and Applied Math. 1. 32.

angle. Hence E is in the circumference PFC, and the circumference CFP is the reciprocal of the straight line AB.

Cor. 1. A straight line through P is reciprocal to itself.

Cor. 2. Circumferences reciprocal to parallel straight lines are tangent to each other at the pole.

The reciprocal to the circumference of a circle is a circumference.

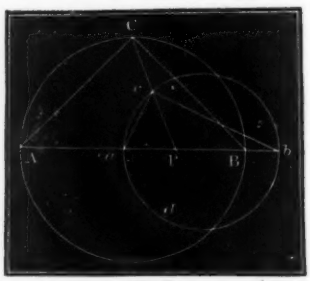


Fig. 2.

Let ABC (Fig. 2) be a circumference, and BA the diameter through P. Take a and b points reciprocal to A and B, and on ab as a diameter describe the circle adb. These two circumferences are reciprocal to each other. For let C be any point of ABC, and c its reciprocal point. Join CP, CA, CB, ca, and cb. Since $PB \cdot Pb = PC \cdot Pc$ the triangles PBC and Pbc are similar, and the angle $PBC = Pcb$. For a like reason $PAC = Pca$. Therefore $Pcb + Pca$ or acb is equal to $BAC + ABC$ a right angle, and c is in the circumference adb.

If P is without the circle ABC, the demonstration is nearly the same.

If two curves cut each other at any angle, their reciprocals cut each other at an equal angle.

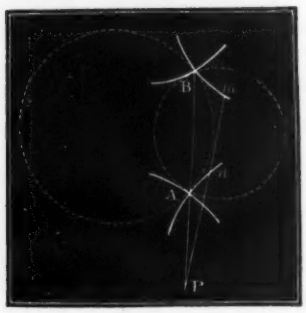


Fig. 3.

Let the two curves (Fig. 3) intersect in A. Their reciprocals will meet in B the reciprocal of A. If n be a point in one curve and m its reciprocal on the reciprocal curve, a circumference may be described through A, B, m, and n, for $PA \cdot PB = Pm \cdot Pn$. If m and n approach B and A and coincide with them, the circle $ABmn$ becomes tangent to one curve at A and to

its reciprocal at B. A second circle may be tangent to the second curve at A and to its reciprocal at B. Now the angle made by these circumferences at A is evidently equal to that at B. Therefore the curves cut each other at the same angle at A as their reciprocals at B.

If now a figure consists of curves and points, and a second figure consists of the reciprocals of these curves and points, and if in the first figure there be proved any proposition or construction, we may, in many cases, infer some corresponding proposition or construction in the second figure.

A few examples will best illustrate this.

All the straight lines that cut a given circumference at right angles pass through the centre.

All circumferences that pass through a given point P and cut a given circumference at right angles, pass through a second common point, which may be called the *reciprocal centre* with respect to P.

All the straight lines that cut a given circumference at the same oblique angle, are tangent to a second circle concentric with the given circle.

All the circumferences that pass through a fixed point P, and cut a given circumference at the same oblique angle, are tangent to a circle.

Two straight lines may be drawn parallel to a given straight line, and tangent to a given circle.

Two circles may be drawn to touch a given circle at a given point P, and also tangent to a second given circle.

To construct the points of tangency, draw through the centre of the given circle a straight line at right angles to the given straight line. It will meet the circumference in the required points.

To construct the points of tangency, draw through the given point P and the reciprocal centre of the second circle with respect to P, a circumference cutting the first circumference at right angles. It will meet the second circumference in the required points.

If through three points A, B, and C, taken two and two, three straight lines, AB, BC, and AC, and also the circle ABC be drawn, then will any one of these straight lines cut the circumference ABC at an

If through a point P and through three points A, B, and C, taken two and two, three circumferences PAB, PBC, and PAC and also the circumference ABC be drawn, then will any one of the circum-

angle equal to that made by the other two straight lines. — (Euc. 32. 3.)

ferences that pass through P cut ABC at an angle equal to that made by the other two circumferences.

This is the fourth prize problem in the Jan. No. of the Monthly.

If three circles be given, two of which intersect, and one of the points of intersection be taken as pole, the reciprocal figure will consist of two straight lines and a circle. To draw a circle tangent to three given circles, corresponds to drawing a circle tangent to two given straight lines and a given circle.

We may do this by the aid of the following Lemma.

LEMMA.

If in any plane there be given two straight lines and a circle, and if about the circle there be described a parallelogram whose sides are parallel to the two given straight lines, and if four straight lines be drawn from the four angles of this parallelogram to the intersection of the given straight lines, then through each point where any one of these four lines cuts the circumference of the given circle, can a circle be described to touch both the given straight lines and the given circle.

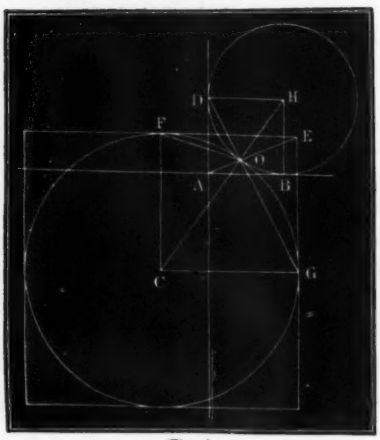


Fig. 4.

Let C (Fig. 4) be the centre of the given circle, AB and AD the given straight lines, EF and EG sides of the parallelogram parallel respectively to AB and AD, and touching the circle in F and G. Draw AE cutting the circumference of the circle in O, GO cutting AD in D, FO cutting AB in B, BH perpendicular to AB and meeting a line through C and O in H. Join CF, CG, and DH. Since EG is

parallel to AD, EF to AB, and CF to BH, we have the proportions $GO : OD :: EO : OA :: FO : OB :: CO : OH$. Therefore CG and DH are parallel, and DH is perpendicular to AD. Likewise we have $CF : CO : CG :: HB : HO : HD$. But the first three terms being equal the last three are equal to each other, and a circle described from H as centre with a radius HB will pass through B, D, and O, and will evidently touch both the lines and the circle in these points.

It may be easily shown, that through no point of the circumference of the given circle other than those on the four lines drawn as above can a circle be described to touch both the given lines and the given circle.

We may now construct the points of tangency of all the circles that can touch two given straight lines and a given circle, and hence also the points of tangency of all the circles which touch three given circles, two of which must, however, intersect.

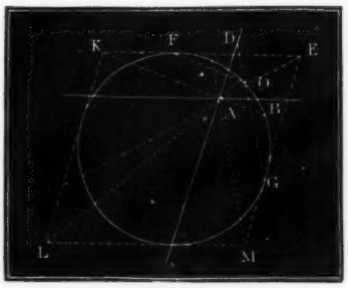


Fig. 5.

Let AB and AD (Fig. 5) be the two straight lines, and GOF the given circle. Draw two straight lines parallel to the straight line AB touching the circle GOF; also two parallel to AD and tangent to GOF. Through A and each one of the four points of intersection, E, K, L,

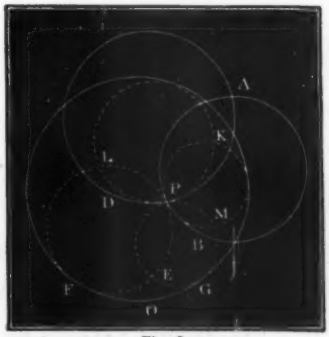


Fig. 6.

Let PAB and PAD (Fig. 6) be the two circles which intersect and GOF the third given circle. Draw two circles touching the circle PAB in P and touching also the circle GOF; also two circles touching PAD in P and tangent to GOF. Through P and A, and each one of the

and M, of these tangents draw straight lines. The points where they cut the circumference $G\,O\,F$ are the points where the required circles touch it. The points of tangency on $A\,B$ and $A\,D$ may be found thus. Let the straight line $A\,E$ determine a point of tangency O on $G\,O\,F$, and let the line through E parallel to $A\,D$ and tangent to $G\,O\,F$ touch $G\,O\,F$ in G. Draw a straight line through G and O, cutting the line $A\,D$ in D. In like manner determine B on $A\,B$. The circle described through D, B, and O will touch in those points the given straight lines and the given circle.

four points of intersection, E, K, L, and M of these circles, describe circles. The points where they cut the circumference $G\,O\,F$ are the points where the required circles touch it. The points of tangency on the circles $P\,A\,B$ and $P\,A\,D$ may be found thus. Let the circle $P\,A\,E$ determine a point of tangency O on $G\,O\,F$, and let the circle through E tangent to $P\,A\,D$ at P and tangent to $G\,O\,F$ touch $G\,O\,F$ in G. Describe a circle through P, G, and O, cutting the circle $P\,A\,D$ in D. In like manner determine B on $P\,A\,B$. The circle described through D, B, and O will touch the three given circles. If one or more of the three given circles were replaced by straight lines this construction would with slight changes be applicable.

THE MOST THOROUGH UNIFORM DISTRIBUTION OF POINTS ABOUT AN AXIS.

By Chauncey Wright, Nautical Almanac Office, Cambridge, Mass.

Let it be required to place at equal successive intervals round and round upon the circumference of a circle, an indefinite number of points, so that the circumference shall at any time be divided by them into the smallest parts; that is, so that the circumference shall be most thoroughly divided at every step of the distribution.

If we were not limited to equal intervals between the successive points, the distribution might be very simply effected either by continually bisecting or trisecting the parts of the circumference. Thus two points placed oppositely would divide the circumference into two equal parts, and with two other points these semi-circumferences might be bisected, according to the arrangement of cruciform flowers

and whorls; and further, four more points might bisect the quadrants, and so on. Or, again, the circumference might at first be divided by three points into three equal parts, and these might be bisected or trisected by the three or six following points, and so on.

To divide the parts of the circumference into smaller fractions than thirds would be to neglect the distribution at first, though the ultimate division of the circumference would be quite as perfect.

But if now we seek a uniform and symmetrical distribution as well as a thorough one, the interval between the successive points must be constant, and if the circumference is to be indefinitely subdivided, this interval is of course incommensurate.

Let x denote the ratio of this interval to the whole circumference; then $\frac{1}{x}$ is the number of times the interval is contained in the whole circumference. Let q denote the integer part of $\frac{1}{x}$, and r' the remainder after subtracting $q\,x$ from the whole circumference.

In the second revolution around the circumference each of the parts x is divided into the parts r' and $x - r'$. In the third revolution each of the parts $x - r'$ into the parts r' and $x - 2\,r'$, and so on as many times as r is contained in x. Let this number be q', and let the remainder, $x - q'\,r'$, be denoted by r''.

By further revolutions each of the intervals r' is subdivided into q'' parts r'' with a third remainder r'''; and so on. The numbers and spaces q, q', q'', &c., r', r'', r''', &c., are such as are obtained by the method for finding the greatest common divisor or for forming a continued fraction, and since

$$1 = q\,x + r', \qquad \frac{1}{x} = q + \frac{r'}{x};$$

$$x = q'\,r' + r'', \qquad \frac{x}{r'} = q' + \frac{r''}{r'};$$

$$r' = q''\,r'' + r''', \qquad \frac{r'}{r''} = q'' + \frac{r'''}{r''};$$

$$r^{[n-1]} = q^{[n]}\,r^{[n]} + r^{[n+1]}, \qquad \frac{r^{[n-1]}}{r^{[n]}} = q^{[n]} + \frac{r^{[n+1]}}{r^{[n]}};$$

we find for x the value $x = \cfrac{1}{q + \cfrac{1}{q' + \cfrac{1}{q'' + \cfrac{1}{q''' + \text{\&c.}}}}}$

Now in order that the circumference may be at every step most thoroughly divided, the magnitudes of the parts into which it is divided should be as nearly equal as possible; that is, the smaller should be contained in the larger the least number of times; hence, in general, the numbers $q^{[n]}$, which express the ratios of successive sub-intervals, should be unity. It may not be necessary that the distribution should begin till after the first revolution. In this case the first quotient q may be any number, but all the other quotients, q', q'', q''', &c., must be unity, hence

(1) $x = \cfrac{1}{q + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \text{\&c., or putting } k = \cfrac{1}{1 + \cfrac{1}{1 + \text{\&c.,}}}}}}}$ we have $x = \frac{1}{q+k}$ in which $k = \frac{1}{1+k}$; whence $k^2 = 1 - k$, or in the form of a proportion $1 : k = k : 1 - k$; that is, k is the ratio of the extreme and mean proportion. Its value is $\frac{1}{2}(\sqrt{5} - 1)$. From the property which it here exhibits we may also call it the *distributive ratio.*

This ratio k and the continued fraction, which expresses it, have hitherto been obtained by mathematical induction from the fractions of the Phyllotaxis. On the other hand we shall be able by further deductions, still subject to the conditions of our problem, to obtain these fractions as special solutions. For, in cases where the circumference is to be divided into a limited number of parts, the interval

x becomes commensurate, and the last remainder or space of the foregoing analysis is contained exactly twice in the last but one; that is, the last step of the distribution consists in bisecting the previous parts of the circumference. Or, what comes to the same, the last remainder but one is contained in the preceding remainder once, with a remainder equal to itself, so that the last two remainders are equal. Hence, for a distributive commensurate interval, all the values $q^{[s]}$ are unity, but are limited in number; and the interval is therefore one of the approximations of the continued fraction (1).

When $q = 1$ these approximations are 1, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{8}$, &c. When $q = 2$ they are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{8}$, &c., the arithmetical complements of the former, and they therefore express the same arrangements, but in an opposite direction around the circumference. When $q = 3$ we obtain $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{7}$, &c.

In general the third approximation $\cfrac{1}{q+\cfrac{1}{1+1}} = \frac{2}{2q+1}$, or two divided by any odd number, expresses the intervals which, in the second revolution, are bisected. The fractions $\frac{2}{3}$, $\frac{2}{5}$, and $\frac{2}{7}$ are the only ones of this simplest form of distribution which are found in the arrangements of leaves around their stems. In all the arrangements of the Phyllotaxis every point after the first revolution is so placed with reference to the two points, between which it falls, that its distance from the middle is never more than one sixth of the whole interval between the two points. The distributive property of these fractions clearly explains their office in nature.

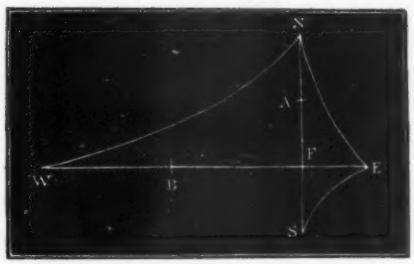
Many other fractions are apparently as well adapted to the symmetry of vegetable forms, and the limitation of natural arrangements to the Phyllotactic system seems, therefore, at first sight, unaccountable. Two hypotheses have been advanced to explain this limitation; the one attempting to deduce these arrangements mechanically from a hypothetical law of formation; and the other

regarding them as typical forms or models which nature has chosen to follow by an arbitrary limitation of her means. The latter hypothesis certainly agrees best with the principles of animal and vegetable Morphology, according to which organized forms are not determined simply by their functions, but are rather certain typical structures or models, modified and metamorphosed by their functions.

There are three ways in which we may seek to account for natural forms and phenomena. First, by deducing them from the law of their development, as in the mechanical sciences; or, secondly, we may account for them by the discovery of their functions or offices as in Physiology; or, thirdly, by referring them to fundamental types or natural methods, as in the Morphology of organized forms. To the latter class of explanations belongs the most generally received account of the Phyllotaxis; that is, the arrangements of this system are regarded as the models by which nature works, and not as the result of any discoverable law of formation, nor as performing any special function in the vegetable economy. But we have shown that there is a purpose which these arrangements fulfil, and that no other arrangements are adapted to the same office. This office in plants is to effect the most thorough distribution of leaves and petals around their axes; to expose them most effectively to the influences of light and air, and to distribute their fibres most thoroughly in the stem. Thus the functions of leaves and petals determine not only their general forms, but also in general their relative positions. Expansion and exposure are the conditions required by the functions, and the Phyllotaxis is only a simple mathematical deduction from these conditions.

PROBLEMS IN "CURVES OF PURSUIT."

1. Three foxes are supposed to be at F, and a dog at S, forty rods south of F. They all start at the same instant, the foxes running with the same uniform velocity and the dog just twice as fast. The first fox runs east along the line FE; the second one runs north along the line FN; and the third runs west along the line FW. The dog runs directly towards the first fox, and overtakes it at E. He immediately pursues the second one and overtakes it at N; then pursues the third and overtakes it at W. How far must the dog run to catch the three foxes? — Communicated by Professor Root.



The general problem, when the body pursued moves on a straight line, may be stated thus: Find the curve, such, that the intercepts of its tangents on the axis of x shall always bear a constant ratio to the lengths of the arcs between the corresponding points of tangency.

2. The problem is still more general when the body pursued moves on a given curve instead of a straight line. It may be thus stated: Find the curve, such, that the intercepts of its tangents on a given curve shall always bear a constant ratio to the lengths of its arcs between the corresponding points of tangency.

When the given curve is a circle, and the required curve starts at the centre, and the ratio is unity, we have a special case which was communicated by Joseph Ficklin, Jr., Esq.

If A pursues B, and their velocities are in the ratio of m to n, there are obviously three cases to be considered. First, when $m > n$, A will overtake B, and in the complete discussion A must pass and

afterwards move directly away from B. Second, when $m = n$, A will never overtake B, and the paths will ultimately become asymptotes to each other, except in those cases where A must continue to cross and recross B's path. Third, when $m < n$, A will never overtake B; but its nearest approach is a point to be determined.

The class of curves known as "curves of pursuit" were named and first discussed by BOUGUER, in a Memoir, entitled "*Upon new Curves to which we may give the name of Lines of Pursuit,*" and printed among the *Mémoires de l'Académie Royale des Sciences,* for the year 1732.

M. BOUGUER remarks, that besides the general interest which should attach to these curves as a class possessing a characteristic property, they are frequently traced, and especially at sea, where the erroneous practice prevails of always directing the prow of the vessel towards the one to which chase is given; thus sailing on the curve of pursuit instead of the straight lines of shortest distance, which will always be directed ahead of the pursued vessel. He only discusses the case in which the pursued body moves on a straight line, not referring at all to the more general problem. The Memoir immediately following that of M. BOUGUER, in the same volume, is by the illustrious M. DE MAUPERTUIS, who solves the problem as announced by BOUGUER, then states the more general one, and gives a general method for finding its differential equation; which, however, he does not integrate for any special cases.

Nor am I aware that the equation has ever been integrated even in the simplest case, that in which the given curve is a circle. When, however, the velocities are equal, as in Mr. FICKLIN'S problem, the curve of pursuit continually approaches the circle, to which it becomes an asymptote, and meets only after an infinite number of revolutions, when the two bodies will be together. When $m > n$, A will overtake B, and if A's motion continues, its path outside of the

circumference will be a wave-like curve, the oscillations growing smaller as *A*'s distance from the circumference becomes greater.

In Vol. II. of the *Correspondance sur l'école polytechnique*, p. 275, we find the following statement:

"M. Dubois-Aymé se promenait sur le bord de la mer; il aperçut, à quelque distance, quelqu'un de sa connaissance, et se mit à courir pour l'atteindre; son chien, qui s'était écarté, courut vers lui, en décrivant une courbe dont l'empreinte resta sur le sable. M. Dubois, revenant sur ses pas, fut frappé de la régularité de cette courbe, et il en chercha l'équation, en supposant, 1.° que le chien se dirigeait constamment vers l'endroit où il voyait son maître; 2.° que le maître parcourait une ligne droite; 3.° que les vitesses du maître et du chien étaient uniformes."

In connection with the above, an erroneous equation of the curve is given. The next reference to the problem may be found in Gergonne's *Annales de Mathématiques*, Vol. 13, p. 145, in a Memoir entitled *Solution Nouvelle d'un problème énoncé dans la correspondance sur l'école polytechnique;* par M. Thomas de St-Laurent.

As this title indicates, there is no reference in this Memoir to the discussion of the same problem made nearly one hundred years before; and as it is called *Problème du chien*, it is to be supposed that the author did not even know the problem by its earlier name. In the same volume of Gergonne's *Annales*, p. 289, M. Thomas de St-Laurent and M. Ch. Sturm have solved the following problem, which is an extension of the one quoted above:

"Problème. Un chien, qui se trouve en un point donnè de l'un des bords d'un canal rectiligne d'une largeur constante, apercevant, en un point donné de l'autre bord, son maître qui marche le long de ce bord, avec une vitesse constante, se jette à la nage pour le joindre. En nageant, il se dirige constamment vers son maître, avec un effort toujours constant; mais le courant de l'eau, en l'entraînant, le détourne sans cesse, et avec un effort également constant, de la direction qu'il veut prendre; on demande, d'après ces diverses circonstances, quelle courbe ce chien décrira sur la surface de l'eau?"

In the Table of Contents these problems are called Problems in "Curves of Pursuit."

A SECOND BOOK IN GEOMETRY.

REASONING UPON FACTS.

BY THOMAS HILL.

PREFACE.

THIS book is intended as a sequel to the "First Lessons in Geometry," and, therefore, presupposes some acquaintance with that little treatise. I think it better, however, that some interval should elapse between the study of that book and of this, — during which time the child may be occupied in the study of Arithmetic.

Geometrical facts and conceptions are easier to a child than those of Arithmetic, but arithmetical reasoning is easier than geometrical. The true scientific order in a mathematical education would therefore be, to begin with the facts of Geometry, then take both the facts and reasoning of Arithmetic and afterwards return to Geometry, not to its facts only, but to its proofs.

The object of "First Lessons in Geometry" is to develop the child's powers of conception; the object of this book is to develop his powers of reasoning. That book I consider adapted to children from six to twelve years of age, this to children from twelve to eighteen years old.

CHAPTER I.

PRELIMINARY.

1. GEOMETRY is the science of form. We really begin to learn Geometry when we first begin to notice the forms of things about us. Some persons observe forms much more closely than others do; partly owing to their natural taste, and partly to their peculiar education. The study of plants, animals, and minerals, the practice of drawing, and the use of building blocks and geometrical puzzles, are good modes of leading one to notice quickly and accurately differences of form.

2. The second step in learning Geometry is to become able to imagine perfect forms, without seeing them drawn. The little book called "First Lessons in Geometry" was chiefly designed to help in the attainment of this power. It is filled with descriptions of forms that cannot be exactly drawn. This is especially true of many of the curves, which cannot be drawn so exactly as straight-lined figures and circles; but which we can, with equal ease, imagine perfect.

3. The third step in learning Geometry is to learn to reason about forms, and to prove the truth of the interesting facts that we have observed. This is the only way in which we can become able to find out new truths and to be certain that they are true. And the first part of this second book is written to teach the scholar how to reason out or prove geometrical truths.

4. After learning to reason out or prove geometrical truths, it is pleasant to know how to use them. This is not the only object of Geometry. It is worth while to know a truth, simply because it is true. But it is also pleasant to be able to apply that truth to practical use, for the benefit of our fellow men. And the second part of this book is written to show in what way we can turn Geometry to practical use.

CHAPTER II.

DEFINITIONS.

5. GEOMETRY is the science of form. Every form or shape is, in general, enclosed by a surface; every surface can be imagined as bounded, or else as divided by lines; and in every line we can imagine an endless number of points.

6. A point is a place without any size. It has a position, but no dimensions; neither length, breadth, nor depth.

7. A line is a place having length, without breadth or depth. As we attempt to mark the position of a point by making a dot with the point of a pen or pencil, and the position of a line by moving the pencil point along the surface of the paper, we find it convenient to speak of a geometrical line as if it were made by the motion of a geometrical point. As the eye runs along the pencil line, so the eye of the mind runs along the geometrical line from point to point.

8. A surface is a place having length and breadth, without depth.

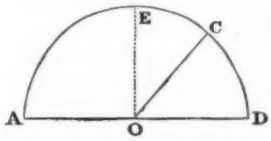
9. A solid is a place having length, breadth, and depth. A geometrical solid is not a solid body, but is simply the space that a solid body would occupy, if it were of that shape and in that place. In like manner a geometrical surface is not the surface of a solid body, but simply the surface of a geometrical solid.

10. A straight line is a line that does not bend in any part. A point moving in it never changes the direction of its motion, unless it reverses its direction.

11. A curve is a line that bends very slightly at every point. It must not have any straight portion, nor any corners; that is to say, it must bend at every point, but the bend must be too small, at each place, to be perceptible.

12. A plane is a geometrical surface, such that a point, moving in a straight line from any one point in the surface to any other point, never leaves the surface. The common name of a plane is "a *flat* surface."

13. An angle is the difference of two directions in one plane. If the line C O should turn around the point O so as to make the arc D C grow larger, the difference of the directions of O C and O D would increase, and we should say that the angle D O C grew larger and larger until the point C arrived at A, so that the two lines O D and O C were opposite in direction.



14. If the point C were carried round half way to the opposite point A, that is, to the point E, the angle D O C would be a right angle, as D O E is. A right angle is a difference of direction half as large as that of oppositeness of direction. The difference between an angle and a right angle is called the complement of the angle. The difference between an angle and two right angles is called the supplement of the angle. Thus C O E is the complement of D O C, and C O A is the supplement of D O C.

15. When two lines make no angle with each other or make two right angles, they are called parallel lines. That is to say, parallel lines are those that lie in the same direction or in opposite directions. When two lines in a plane are not parallel, the point where they cross, or would cross if prolonged, is called the vertex of the angle.

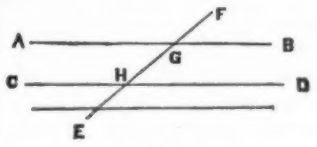
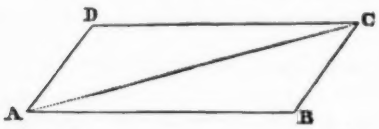


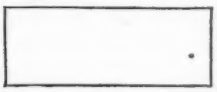
Fig. A.

16. A triangle is a figure inclosed by three straight lines in one plane.

17. A right triangle is a triangle in which two of the sides make a right angle with each other. These sides are then called the legs of the triangle, while the third side is called the hypothenuse.

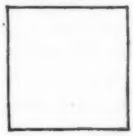


18. A parallelogram is a figure bounded by four straight lines in a plane, with its opposite sides parallel.



19. A rectangle is a parallelogram with its angles all right angles.

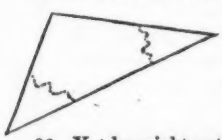
20. A square is a rectangle with its sides all equal.



CHAPTER III.

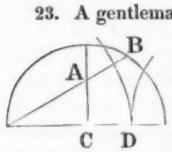
REASONING.

21. Suppose that we wished to make another person believe that the three angles of a triangle are, together, equal to two right angles. One way of convincing him would be to take a triangular piece of card, or of paper, cut off the corners by a waving line, and lay the three corners together, to show him that the outer edges will make a straight line, as two square corners put together will do.



22. Yet he might not be satisfied that the line was perfectly straight. Or perhaps he might say that if the angles of the triangle were in a different proportion, the corners put together would not make a straight line with their outer edges.

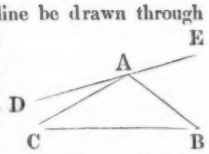
23. A gentleman once came to me and said, "I have found out that if you draw such and such lines, you will always find these two, A B and C D, equal. At least my most careful measurement shows no difference between them." I said to another gentleman, who knew something of Geometry, "Can you prove that these lines will be equal if the figure is drawn exactly as directed?" He said he would try, and in a few days he sent me what he called a proof. But on reading it I found it only amounted to saying that "if the lines are equal, they are equal." I then examined the matter myself and found that the lines were, in reality, never equal, although the difference was always very small, — too small to be easily discovered by measurement.



24. Such errors, too small to be discovered by measurement, are sometimes large enough to do great mischief; and at any rate, however small, they are still errors, and it is best to get rid of errors, and to find the exact truth, whether the error is mischievous or not. In order to do this we must learn how to reason, how to prove truths. And in order to avoid such mistakes as that of my friend, who thought he had proved the false proposition of which I have been speaking, we must learn to reason correctly.

25. When we put the corners of a paper triangle together to make a straight line, we may say, Perhaps there is some slight error here, too small to be detected by measurement. How then shall we prove that there is no such error in a perfect geometrical triangle?

26. The first thought that occurs to us will be, that if any straight line be drawn through one vertex of a triangle, as D E is drawn through the point A, without passing through the triangle, the three angles on one side of the line, about the point A are equal to two right angles, and if the sum of the three angles of the triangle is equal to two right angles, it must be equal to that of the three about the point A.



27. But as the central angle at A is already an angle of the triangle, it follows that the other two angles must be equal in their sum to the sum of the angles B and C.

28. Now this will be true in whatever direction the line D E is drawn, only provided it does not pass through the triangle. Let us then imagine it to pass in such a direction as to make the angle B A E equal to the angle A B C and it will only be necessary to prove that D A C is then equal to A C B. For if D A C is equal to A C B, then since we suppose B A E equal to A B C, and B A C is one of the angles of the triangle, we shall have the three angles about A equal to the three angles of the triangle, and as the three angles about A are equal in their sum to two right angles, the three angles of the triangle will be equal to two right angles, which is what we wish to prove.

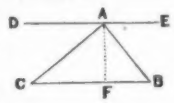


Fig. B.

29. But to say that D A C is equal to A C B is equivalent to saying that A D and C B differ equally in their direction from the direction of A C, and since A C is a straight line and its direction from A is opposite to its direction from C, this is equivalent to saying that A D and C B go in opposite directions.

30. All that we have now to prove is, that the line A D or E D goes in the same direction as the line B C. But this needs no proof, because we have already supposed that E A makes the same angle with A B that B C does; and as B C is a straight line, the direction of E A and C B must be opposite. But as E A is part of the same straight line with A D, it has the same direction as A D. The proof is now complete.

31. And this mode of proof does not depend at all upon the particular shape of the triangle. We have made no supposition concerning the shape of A B C, except that it should be a triangle. We have, therefore, proved that the sum of the three angles of any triangle is equivalent to two right angles.

32. Thus we have analyzed the proposition that the sum of the three angles of a triangle is equivalent to two right angles, and found that it resolved itself at last into saying that two lines making equal angles on opposite sides at the end of a straight line must point in opposite directions, a proposition which is easily shown to be true.

33. But this mode of analyzing is very tedious when stated in words. A geometer usually does not state it; he passes through it very rapidly in his own mind, and then restates the process carefully in an inverted order, as follows in articles 34, 35, and 36.

34. When one straight line crosses another, the opposite or vertical angles are equal. For since each line has but one direction, the difference of direction on one side of the vertex must be the same as on the other side.

35. When one straight line crosses two parallel straight lines, the alternate internal angles are equal, or in the figure (Fig. A.) A G E is equal to D H F. For D H F is equal to B G F, having its sides pointing in the same direction as those of B G F, and A G E is equal to B G F by article 34.

36. Through the vertex of any triangle, as through A (Fig. B.), draw a straight line D E parallel to the opposite side B C. Then E A B will be equal to its alternate internal angle A B C, and for the same reason D A C will be equal to A C B. So that the three angles of

the triangle will be equal to the three angles about the point A, and their sum is plainly two right angles.

37. The mode of proving that the sum of the three angles of a triangle is equal to two right angles, by cutting a piece of card, is called experimental proof. It is of very little use in mathematics, but of great use in the study of physics, especially in mechanics and chemistry.

38. The mode of proof used in articles 26–31 is called, by metaphysicians and by writers on Arithmetic, *analysis*. But as geometers, in their writings, almost never use this method, they have no name for it; and when they speak of analysis or of analytical methods they usually refer to something else of a very different character.

39. The mode of proof in articles 34–36, called by metaphysicians, synthesis, by geometers, demonstration or deduction, is that usually employed in stating geometrical results. This mode is chiefly applicable to mathematics, and must be used with very great caution in reasoning upon other subjects.

40. A proposition which we wish to prove may be compared to a mountain peak which we wish to show is accessible from the highway. The method of articles 26–31 may be compared to taking a flight by a balloon to the top of the peak and then finding a path down to the highway; while the method of articles 34–36 may be compared to the direct ascent of the mountain. In either case we show that the peak is accessible; because we actually pass over all the steps of a connected pathway between the road and the mountain top.

Thus in geometrical demonstration we pass through every step connecting the simplest self-evident truths with the highest deductions of the science; while in the process which writers on Arithmetic call analysis, we pass over every step from those truths down to the simplest. In either case we prove that the higher truth really stands on the same basis as the simpler, and must, therefore, be true.

See page three of Cover.

Editorial Items.

The following gentlemen have sent us solutions of the Prize Problems in the January number of the Monthly:

Wm. E. Merrill, Cadet, 1st Class, U. S. Military Academy, answered all the questions. (A. E. Church, Prof.).

Charles Bettle, Sophomore Class, Haverford College, Pa., answered all but IV. and V. (M. C. Stevens, Prof.).

Prof. Stevens also communicated a very elegant solution of problem IV. by Wm. G. Rhoads, Esq., graduate of Haverford College, Class of 1858.

David Trowbridge, Perry City, Schuyler Co., N. Y., answered problems III. and V.

W. F. Osborne, Sophomore Class, Wesleyan University, Middletown, Ct., answered all but V.

Asher B. Evans, Junior Class, Madison University, Hamilton, N. Y., answered all the questions.

We shall publish the report of the Judges, with the Prize Solutions, in the next number.

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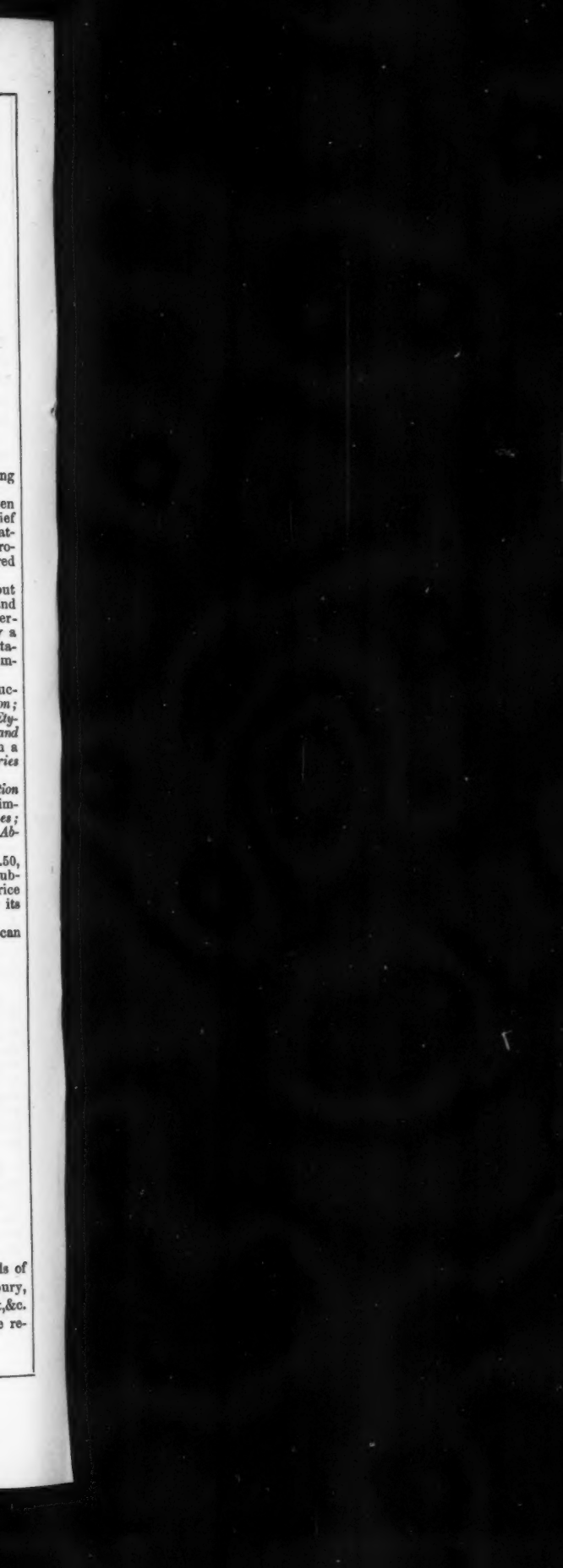

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The above Table of Contents will give Teachers and Students a very good idea of Mr. HILL's "Second Book in Geometry," begun in this number of the Monthly. The problems and undemonstrated theorems comprising Part III. instead of being given together, will be introduced just as fast as the student has gone over sufficient ground to understand them. It will be the aim of the author to furnish an elementary work embracing all the essential ideas "On the Study of Geometry," so ably set forth in former numbers of this Journal. We hope teachers will put this book into the hands of pupils just commencing the study of Geometry, in order fully to test the question whether they can learn to invent demonstrations in Geometry with the same facility as they do solutions in Arithmetic and Algebra. We are indebted to Messrs. HICKLING, SWAN & BREWER for most of the cuts to be used in the volume.

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